RP 52

AN INTERFERENCE METHOD FOR THE DETERMINATION OF AXIAL AND OBLIQUE ABERRATIONS

By A. H. Bennett

ABSTRACT

An interference method for the determination of the axial and oblique monochromatic aberrations of a lens system is described. Equations necessary in the design of the apparatus, and for the computation of the results are given. The simplicity of the apparatus, especially the use of but one auxiliary reflecting surface, lessens the probability of large instrumental errors. The aberrations of three astronomical objectives are investigated, and the results are expressed in the form of phase contours.

CONTENTS

I. INTRODUCTION

In a paper on a modification of the Hartmann [1] test, based on interference, Gardner and Bennett [2] have described a method for the determination of the spherical aberration of a lens system. The following brief review of its essential characteristics is given, since the method described in this paper is a development along similar lines of procedure.

If the small apertures used in isolating the rays in the Hartmann method are correctly spaced and of suitable diameter, interference

[1] Hartmann, Zeits. f. Instrumentenk., **24**, p. 1; 1904.

[2] I. C. Gardner and A. H. Bennett, J. Opt. Soc. Am. and Rev. Sci. Inst., **11**, pp. 441-452; 1925. Also Trans. by H. Kessler, Zeits. f. Instrumentenk., **47**, pp. 197-204; 1927, and Geiger and Scheel's Handb. d. Phys., **18**, pp. 810-811.

fringes are formed between adjacent extra-focal images of these apertures. Advantage is taken of the displacement of the fringes caused by differential aberration between adjacent apertures. By adding these differential aberrations, determined by a measurement of the displacement of the fringes, the total phase aberration is found. This method combines many of the desirable features of several prior methods. It requires no more elaborate apparatus than that used in the original Hartmann method. High precision is obtainable because the measurements are made on narrow interference fringes rather than on the conventional extra and infra focal shadowgraph disks, which are always of larger diameter than is desirable. In this respect the method is similar to that devised by Merland.[3] As in the interference methods of Twyman,[4] Waetzman and Bratke,[5] Michelson,[6] and Cotton,[7] the magnitudes of the aberrations are expressed as phase differences rather than as geometrical aberrations.

In the present paper the Gardner-Bennett method outlined above has been extended to apply to a wave front proceeding from an object point, either on the axis or at some distance from the center of the field. Measurements can be made along any desired number of meridians, and the aberration can be determined at a sufficient number of points to enable a contour map to be drawn showing the form of the entire wave surface which emerges from the lens. These contours are similar in many respects to the Twyman[8] interferograms.

The applications of the method herein described have been restricted to the measurement of the monochromatic aberrations of telescope objectives. In view of the small magnitudes of the aberrations present in a well-designed and constructed astronomical telescope objective, the application of the method to such a lens should serve to demonstrate the suitability of the procedure when applied to microscope or photographic objectives or other types of lenses. A further reason for selecting telescope objectives was the need of information concerning the amounts of axial and oblique aberration of three astronomical objectives of 5½ to 12 inches diameter and having focal lengths from 6 to 12 feet, which are a part of the laboratory equipment of the optical instrument section of the Bureau of Standards. The results of the measurements described in this paper are of interest beyond their indications concerning the adaptability of the objectives to their special purposes because they also show particular effects arising from certain characteristics of design, materials, and construction.

---

[3] A. Merland, Rev. d'Opt., **3**, pp. 401–413; 1924.
[4] F. Twyman, Phil. Mag. (6), **35**, p. 49; 1918.
[5] Bratke and Waetzman, Ann. d Phys., 72, **7**, pp. 501–515; 1923.
[6] A. Michelson, Astrophys. J., **47**, pp. 283–288; 1918.
[7] A. Cotton, Physica, **1**, pp. 274–283; 1921.
[8] See footnote 4.

II. DESCRIPTION OF METHOD

1. THEORY

If a series of equidistant slits of equal width are placed within or beyond the focal point of a lens, the relation between slit width and distances between centers can be made such that the separation of the geometric bundles of rays at some position along their course will permit the first diffraction bands from any pair of adjacent slits to overlap. Over the region of overlapping a group of sharp fringes[9] is visible provided the source of light is of small dimensions. It was shown in the previously mentioned paper that when the grating is placed within the focus the position of the central fringe in each group is dependent on the shape of the portion of the emergent wave front included between the two corresponding adjacent slits. That the same condition obtains when the series of slits is placed beyond the focal point can be shown in a similar manner, as follows:

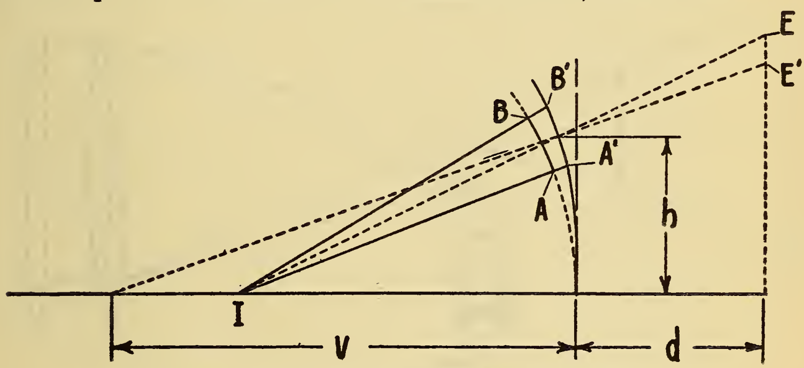


FIG. 1.—*Sketch illustrating theory of method*

In Figure 1 let I be the focus through which the waves are assumed to pass, proceeding from left to right, before they are transmitted by the diaphragm. A and B are the portions of a spherical wave front, with its center at I, which are transmitted through two adjacent apertures. A' and B' represent the corresponding portions of the actual wave front distorted by the presence of aberrations. The problem at hand is to find the difference in phase between the distorted and ideal wave fronts in passing through I, the selected focus. In the case illustrated the marginal portions at A' or B' have been accelerated with respect to the axial portion which is tangent to the ideal spherical wave front. The phase acceleration on arrival at I is measured by the radial distance between the two wave fronts. Portions of the spherical wave front passing through the two adjacent apertures would interfere and form a central fringe at E on the radius passing midway between A and B. The actual

[9] R. W. Wood, Physical Optics, p. 190.

distorted wave front likewise forms a central fringe at E', which is on the perpendicular bisector of the chord $A' B'$. The phase difference for points A' and B' on the actual wave front on arrival at I is

$$\frac{A\,A' - B\,B'}{\lambda}$$

Using w to represent the distance between two adjacent fringes in a group on the plane at $E\,E'$, the quantity $\frac{E\,E'}{w}$ is likewise the phase difference at the focus I in wave lengths between the portions of the wave transmitted by the two adjacent apertures.
Hence

$$\frac{A\,A' - B\,B'}{\lambda} = \frac{E\,E'}{w} \tag{1}$$

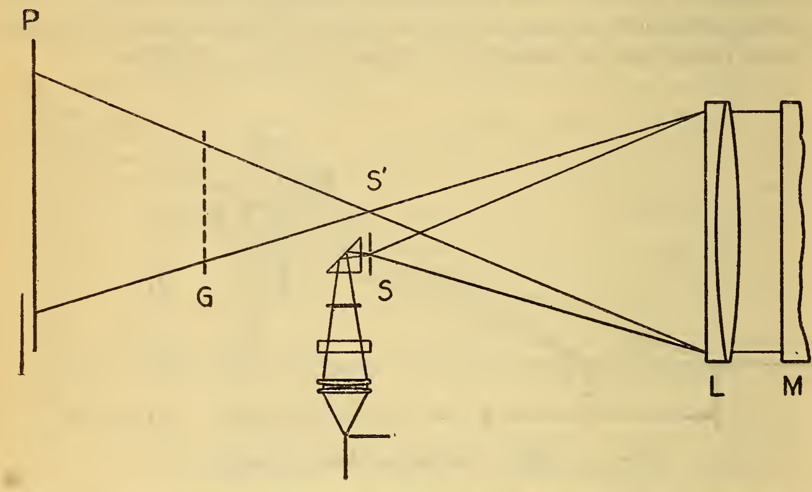


FIG. 2.—*Diagram of apparatus*

The phase difference between an axial point on the wave front and a point at any height can be found by adding the consecutive values of $\frac{E\,E'}{w}$ for each pair of apertures.

2. APPARATUS

A diagram (not to scale) of the apparatus is shown in Figure 2. Light from a carbon arc, after passing through a water cell and a Wratten No. 73 "monochromatic" filter having its maximum transmission at about 580 mμ, is focused through a total reflecting prism on the small hole S, of which the diameter is approximately 0.1 mm. This artificial star is placed about 2 mm to one side of the axis of the telescope objective L, under test. The mirror M is a front silvered glass flat, whose diameter should be as large as that of the lens. After reflection at M the light is focused at S'. For the determination of the axial aberration the entire system was aligned on a very

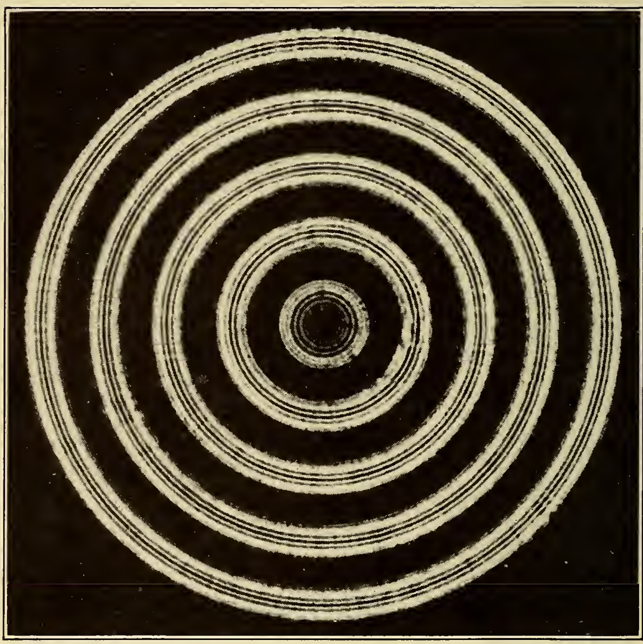


FIG. 4.—*Interference fringes*

This illustration is an enlarged negative of only the central portion of an entire fringe system.

rigid optical bench so that the "point" source S and its image S' were symmetrically placed with respect to the axis of the lens. For the oblique measurements the source S was kept in the position indicated, and the mirror M was tilted to throw the image S' at the desired oblique position off the axis.

At G is a circular grating, a reproduction of which is shown approximately full size in Figure 3. This grating consists of equispaced, concentric slits in a silver film chemically deposited on glass. The distance between the centers of slits was 1.33 mm, and the slits were 0.30 mm wide. These dimensions were selected in accordance with the relationships expressed in the equations of the next paragraph. The surfaces of the glass plate were flat to about 0.1 λ and parallel to about 1.5 seconds although, as will be shown later, compensation was made for the errors introduced by the glass plate. The fringes formed by the interference of light from consecutive slits are recorded on the photographic plate at P (fig. 2). Several of these fringes are illustrated in Figure 4, which is a negative. In order to avoid fogging the plate by scattered light, the entire apparatus, with the exception of the arc lamp, water cell, and filter, was inclosed.

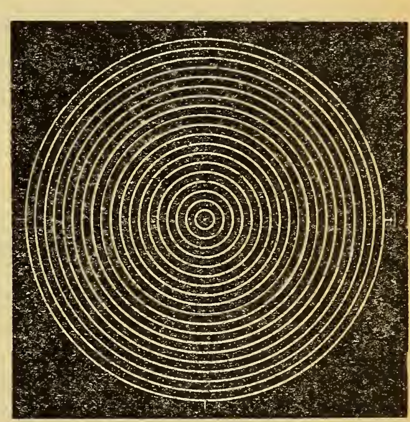


Fig. 3.—*Circular grating* (*full size*)

In the design of the diaphragm for use with this method the correct dimensions must be found if satisfactory fringes are to be obtained on the photographic plate. The fringe system is formed at the overlapping of the first diffraction bands from each pair of adjacent slits. The angle subtended at the diaphragm by the first diffraction band is λ/s, where s is the slit width. At the diaphragm the angular subtense of consecutive minima in the interference system is λ/e, where e is the distance between centers of the slits. Then, if each interference group is to consist of n fringes

$$e = ns \tag{2}$$

In making the measurements of the fringes there should be at least three fringes in a group, since it is necessary to determine the distance between consecutive fringes. It has been found, however, that it is necessary to measure only a few of these values, since they remain practically constant over the photographic plate, provided proper care is used in obtaining uniform spacing of the concentric slits on the diaphragm. In equation (2) it is well to set $n = 5$ if three fringes are to be used because of the low intensity of the extreme fringes.

The distance between adjacent fringes is

$$w=\frac{\lambda}{e}d \tag{3}$$

where d is the distance from diaphragm to photographic plate. For overlapping of the first diffraction bands the distance between geometric, extra-focal images must be

$$e'=\frac{3\lambda d}{s}=3wn \tag{4}$$

The distance between diaphragm and intersection of the ray with the axis is

$$V=\frac{ad}{b-a} \tag{5}$$

where a is the distance between the axis and the slit in the diaphragm, and b is the corresponding distance on the photographic plate. If F is the ratio of focal length to aperture of the lens under test and m is the number of central fringes corresponding to the semiaperture, then

$$V=2Fme \tag{6}$$

By combining the above equations (2) to (6) it is found that:

$$e=\frac{6Fmn\lambda w}{w+2Fm\lambda} \tag{7}$$

$$d=\frac{6Fmnw^2}{w+2Fm\lambda} \tag{8}$$

$$b=me' \tag{9}$$

and

$$V=\frac{12F^2m^2n\,\lambda\,w}{w+2Fm\lambda} \tag{10}$$

In designing the apparatus, desired values are assigned to w, the fringe width; s, the slit width; n, the number of fringes in a group; and m, the number of central fringes corresponding to the semiaperture.[10] Then, substituting these values in equations (7), (8), and (10), respectively, e, the distance between centers of slits; d, the distance from focal point to diaphragm; and V, the distance from diaphragm to photographic plate, are computed. The necessary size of the photographic plate is found from equation (9).

Some difficulty was experienced at first in accurately ruling the concentric slits in the silvered surface of the diaphragm (fig. 3). The

[10] As will be shown, this number, m, is the number of points on the graph of phase difference plotted against semiaperture.

final method of construction was as follows: The silvered disk of glass was mounted horizontally on the table of a spectrometer with the silvered surface uppermost. The silvered surface was adjusted normal to the axis of rotation of the conical bearing supporting the table. In order to accomplish this adjustment, an autocollimating telescope was mounted vertically above the surface, and the leveling screws on the spectrometer table were adjusted until the autocollimated image of the reticule of the telescope remained stationary when the table bearing the disk was rotated. After removal of the telescope a micrometer screw, to which was attached a finely pointed German silver stylus, was mounted in a horizontal position above the silvered surface. The correct movement of the stylus was determined by means of the micrometer head, which could be read to 0.001 mm. It was necessary to make a preliminary determination of the width of the cut produced by the stylus, after which, during the process of cutting, it was possible by means of the micrometer screw to move the stylus horizontally by the correct amount to engrave slits of the desired width, having the predetermined distance between centers.

3. EXPERIMENTAL PROCEDURE

(*a*) Adjustment of Apparatus.—As shown in Figure 2, autocollimation was used in the measurements of the phase errors. The axial aberration was first determined by having the source and its image practically on the axis. In measuring the oblique aberration, since the source was kept on the axis, it was necessary to subtract the axial aberration from the measured quantity which consisted of the sum of the true oblique and axial aberrations. Hence, it was necessary to align the apparatus accurately on an optical bench. The location of the axis of the lens under test (fig. 2) was determined by the use of the principle that, if a luminous object is placed on the axis, the object and its images formed by reflection from the surfaces of the lens all lie in a straight line. After the axis was located the mirror M (fig. 2) was adjusted by tilting until its reflecting surface was normal to the axis. With the autocollimating mirror normal to the axis, the object and image become symmetrically placed with respect to the axis. The artificial star S (fig. 2) was adjusted laterally and longitudinally until it was about 4 mm from its image S' in a plane approximately normal to the axis. The diaphragm containing the circular slits and the plate-holding device were constructed as a camera unit. By means of proper adjustment of this camera for height and leveling the diaphragm is made concentric with the cone of rays emerging from the star image. When this condition obtains, the extra-focal images are complete circles.

(*b*) Making the Exposures.—In order to avoid errors arising from tilt of the photographic plate, and from spherical aberration of

the plane parallel plate upon which the concentric slits are ruled, or from irregularities of surfaces and index of refraction of this plate, two photographs were made for each determination of aberration. The first exposure was made with the apparatus adjusted as has just been described. A second exposure was made on another plate with the star image S' replaced by a small aperture about 0.1 mm in diameter. This artificial star was placed in the position previously occupied by the image with the aid of an auxiliary microscope provided with a right-angled objective prism. The same illuminating system was utilized to illuminate this comparison star as was used for the star S (fig. 2). The set of fringes obtained with the pinhole directly represents the ideal condition, since the emerging wave fronts are spherical. The fringes on the first plate are found to be displaced relatively to those on the second. These relative displacements arise principally from the aberration of the test lens. It is necessary, however, to eliminate similar effects resulting from the difference in position of star image and pinhole and the errors of the autocollimating reflecting surface. The method of eliminating these errors is treated later in (*d*) Computation of results.

(*c*) Measurement of the Plates.—In measuring the fringes the object is to determine the distance from the center of the system to the consecutive fringes. It is difficult to have the exact center registered on the photograph. On the first trial a small aperture was placed at the center of the slit system in the diaphragm, but this produced an unsatisfactory center on the photographic plate because of overexposure and halation, as very long exposures are required to show the fainter fringes. The method finally adopted was as follows: A very small hole was made in the emulsion near the center of the finished plate. The positions of the central fringes in each group were measured with respect to this arbitrary point. The average of the readings along each of six diameters was found, and the differences between the average readings and the readings for the arbitrary point were plotted against the azimuths of the diameters. From the maximum displacement along the resulting sine curve and the corresponding azimuth it was possible to locate the true center of the fringe system with reference to the arbitrary center and make proper correction to the readings for the location of the fringes.

(*d*) Computation of Results.—Let the distances from the axis to the fringes on the plate exposed with the test lens in position be a_1, a_2, a_3 . . . . . , and let c_1, c_2, c_3, ... represent the corresponding distances on the plate taken with light emerging directly from the comparison star. Then, a_1-c_1, a_2-c_2 . . . . represent the displacements arising from aberration and, in addition, errors of the reflecting surface and errors of adjustment of the source of the

reference wave front. If w is the distance between two adjacent fringes,

$$\frac{a_1-c_1}{w},\ \frac{a_2-c_2}{w},\ \ldots\ldots\ \frac{a_n-c_n}{w}$$

are the differences in phase between points on the actual wave front corresponding to the respective adjacent apertures in the diaphragm.

Hence, $\sum_1^n \frac{a_n-c_n}{w}=$ the difference in phase, at the nth aperture, between the wave front emerging from the test lens and the spherical reference wave front emerging from the pinhole. As has been pointed out, the reference wave front may have its center displaced laterally or longitudinally from the position of best focus; that is, position of smallest phase residuals, although the error is small by reason of the method used in setting the pinhole to occupy the position of the image. Any error in placing the pinhole may give rise to a tilting and a change in center of the reference wave front. Hence, in computing the results it will generally be necessary to refer the actual wave front to several reference wave fronts having their centers at various positions along the axis (for axial measurements) or the chief ray (for oblique measurements) by which means the effects of a change of focus on the phase residuals are determined. In addition, the reference wave front is tilted until greatest symmetry of phase residuals appears over the entire wave front under investigation. These corrections are made empirically in the following manner.

If $y_1, y_2 \ldots\ldots y_n$ represent the distances from the center of the diaphragm to the respective concentric slits, the effect of a tilt of the reference wave front on the phase difference for different apertures can be expressed as

$$\eta_t = K\, y_1 \sin\psi,\ K\, y_2 \sin\psi,\ \ldots\ldots\ldots \text{ etc.} \tag{11}$$

where K is a constant for all zones of the wave front; that is, all values of y, and ψ is the azimuth of the meridian of the wave front measured from the axis of tilt. The value of K, which is the angle of tilt, and the azimuth of the axis of tilt which furnish the most symmetrical distribution of the phase residuals about the axis are found empirically.

The effect of a change in curvature of the reference wave front on the phase residuals for any aperture is expressed by

$$\eta_f = C\, y_1^2,\ C\, y_2^2,\ \ldots\ldots\ldots\ldots \text{ etc.} \tag{12}$$

where C is a constant for all values of y and is made of such value as to reduce the total phase residuals of the lens to a minimum. The

phase residuals between the actual and ideal wave fronts can now be written [11]

$$\eta=\sum_{1}^{n}\frac{a_n-c_n}{w}+K\ \sin\ \psi\ y_n+C\ y_n^2 \tag{13}$$

where a is the distance from center to fringes on the plate taken with actual wave front, c the corresponding distance on plate taken with the reference wave front, y the distance from center to slit in diaphragm, and K and C the constants of equations (11) and (12).

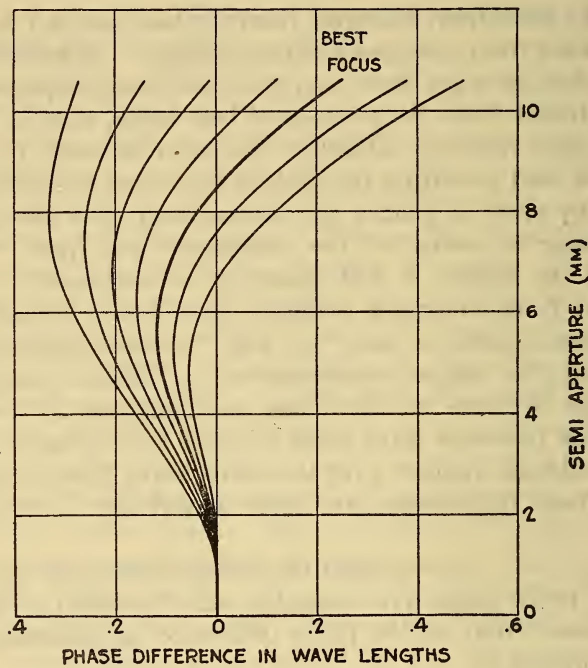


FIG. 5.—*Graphs showing variation in phase differences with change of focus*

In computing the graph marked "best focus" the value of C (equation (13)), which gives smallest phase differences, was used

(*e*) METHOD OF EXPRESSING RESULTS.—The values of η, the phase differences, are plotted against corresponding values of y, the semiapertures of the diaphragm. The latter are closely proportional to the corresponding semiapertures of the objective under test. Figure 5 illustrates a family of such graphs for a single meridian of the wave front. Here various values of the constant C, equation (13), have been applied in order that the phase difference at the best focus can be determined. Such graphs are obtained for each meridian

[11] In equation (13) it should be noted that, by the use of autocollimation, the actual wave front has twice the aberration produced by the lens as it is ordinarily used with incident light in the case of axial imagery. For oblique imagery the axial aberration is also included in the expression for η, the phase difference.

of the wave front measured. From these a set of contours, showing lines of equal phase difference over the entire wave surface can be easily drawn. In most of the examples given meridians at intervals of 30° were measured. As a rule, meridians 30° apart will be sufficient to enable one to gain information concerning the entire wave front, although in cases of irregularity of surfaces of the test lens or lack of homogeneity of the glass it may be desirable to select meridians at smaller intervals.

III. RESULTS

The monochromatic aberrations of three telescope objectives were investigated by the method described. Phase contours,[12] representing the axial and oblique aberrations, were drawn. In all cases the center of the map is taken as the position of zero phase difference, and the numerical values of the contours are indicative of the phase difference expressed in wave lengths ($\lambda = 580$ mμ).

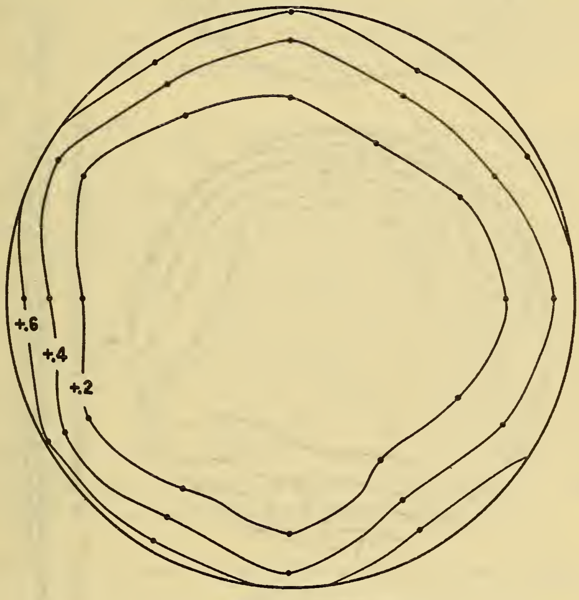


FIG. 6.—*Phase contours for best axial focus of objective No. 1*

1. OBJECTIVE OF 155 mm DIAMETER AND 1,800 mm FOCAL LENGTH (NO. 1)

The contours of Figure 6 show the axial aberration at the best axial focus. The contours are drawn at intervals of 0.2λ. This lens shows a departure of 0.6λ at the edge. However, if the lens is

[12] The characteristic forms of phase contours corresponding to the various geometrical aberrations have been discussed by F. Twyman, Trans. Opt. Soc., London, **22**, pp. 182–185; 1920–21; and also by G. C. Steward, Camb. Phil. Soc. Proc., **24**, pp. 166–170; January, 1928.

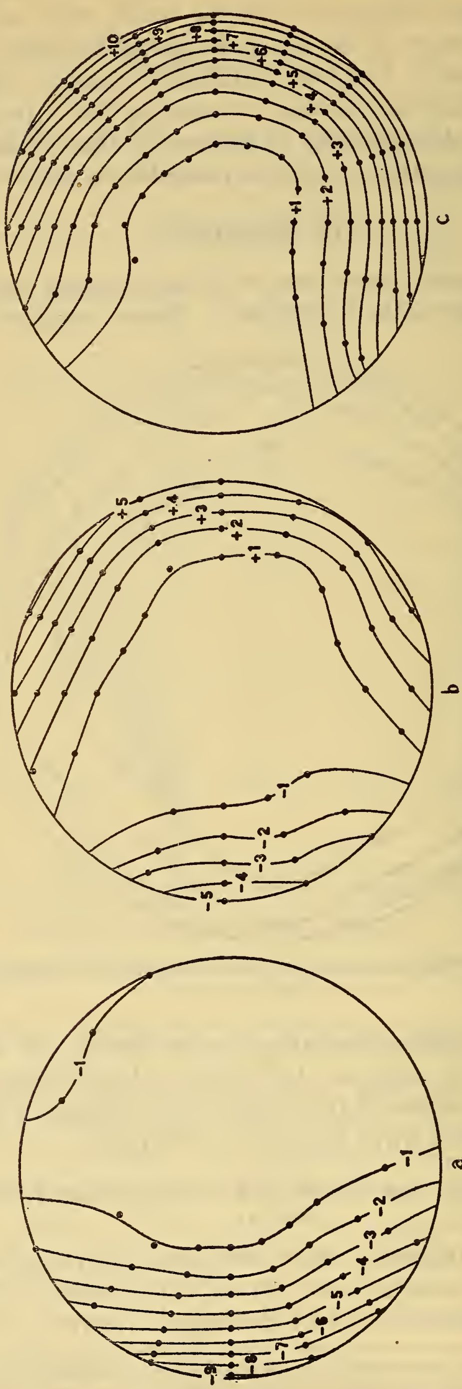


Fig. 7.—*Contours for oblique foci of objective No. 1*

These contours represent conditions at three successive foci along a bundle of rays at an obliquity of 2.5°, *a* and *c* are contours corresponding to positions near the two astigmatic foci. *b* corresponds to the best focus

stopped to about two-thirds of its diameter the Rayleigh tolerance of 0.25λ is not exceeded. The small irregularities in the contours are probably the effects of lack of homogeneity of the glass or errors arising from slight deformation of surfaces. In the apparatus as previously described for testing this lens the mirror M (fig. 2) was tested for flatness by the interferometry section of the Bureau of Standards. The surface was found to be slightly spherical with a sagitta of about 0.1λ. The sphericity of this optical flat does not enter as a correction factor in the results because it simply necessitates a slight amount of refocusing, which is the equivalent of changing the factor C in equation (13). The extremely small aberration produced by this reflecting surface itself is neglected, since it is of a smaller order of magnitude than the actual aberration of the telescope objective under test. No irregularities greater than 0.01λ in the flat were found, hence their effect on the contours could also be neglected.

Figures 7 (*a*), 7 (*b*), and 7 (*c*) show the oblique aberration of this lens for an obliquity of about 2.5° to one side of the axis. Figure 7 (*b*) indicates the phase errors at the best focus for this obliquity. At this angular distance from the axis, which is large for a lens of this type, the definition is seriously impaired by the presence of astigmatism and coma which produce phase errors ranging from +5 to −5 whole wave lengths. Figure 7 (*a*) shows the contours corresponding to one astigmatic focus, and Figure 7 (*c*) shows the other astigmatic focus. These two foci lie on opposite sides of the most nearly symmetrical focus. The asymmetry of the patterns is indicative of a large amount of coma. This is in accordance with the fact that the correction of coma was not sought in the original design of the objective which has an equiconvex crown component.

2. OBJECTIVE OF 145 mm DIAMETER AND 2,900 mm FOCAL LENGTH (NO. 2)

The axial aberration of the above lens is shown in Figure 8. In general, the aberration of the best axial focus does not exceed +0.2 λ, but there are small regions in the lens in which the error reaches a value of −0.2 λ, as shown in the left side of the figure. A consideration of the graphs of phase difference plotted against semiapertures which were made preliminary to the contours showed that these two areas of negative phase difference are not joined. It is believed that these regions represent an actual variation on optical path arising from irregularity of surface or heterogeneity of the glass or, perhaps, a combination of the two.

The oblique aberration at an angular distance of 10 minutes from the axis and at the best oblique focus is shown in Figure 9. The aberration varies from +0.2 λ (over a small portion of the figure)

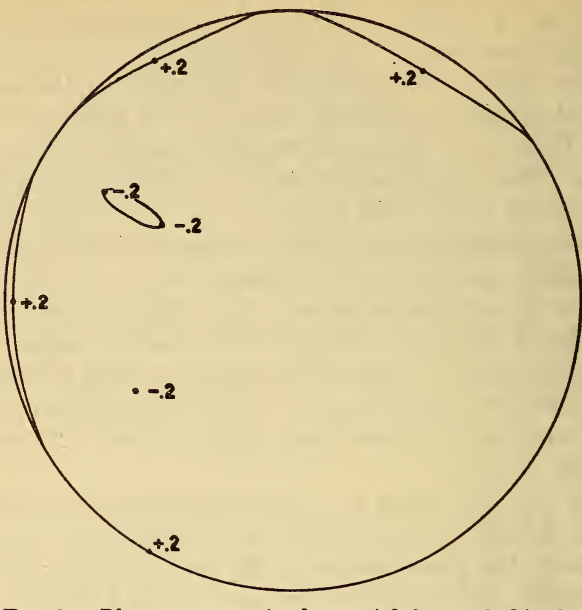


FIG. 8.—*Phase contours for best axial focus of objective No. 2*

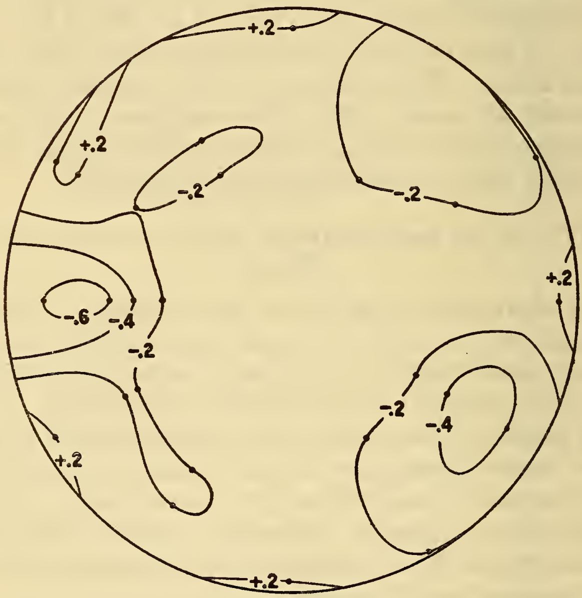


FIG. 9.—*Phase contours for best oblique focus of objective No. 2. (Obliquity 10′)*

to $-0.6\ \lambda$. At this angle the contours show an asymmetry which indicates a radial flare in the image, since the lens is considered to be tilted about a vertical axis. With this objective the same flat was used as with the one previously discussed, and likewise it was not necessary to correct the results for errors introduced by the flat.

3. OBJECTIVE OF 300 mm DIAMETER AND 3,940 mm FOCAL LENGTH (NO. 3)

This objective is an achromatic doublet, corrected for spherical aberration and coma. Although the largest flat available was used for autocollimation, unfortunately it was not of sufficient size to cover the entire aperture of the objective. The aperture covered by the flat was 258 mm in diameter, and the contours of this lens represent conditions over this aperture. The flat was found to be convex but did not sufficiently approach a true sphere, so that the departures from sphericity could be neglected. The flat was measured by the interferometry section of the Bureau of Standards, and the departures from flatness were found to be expressible by the equation

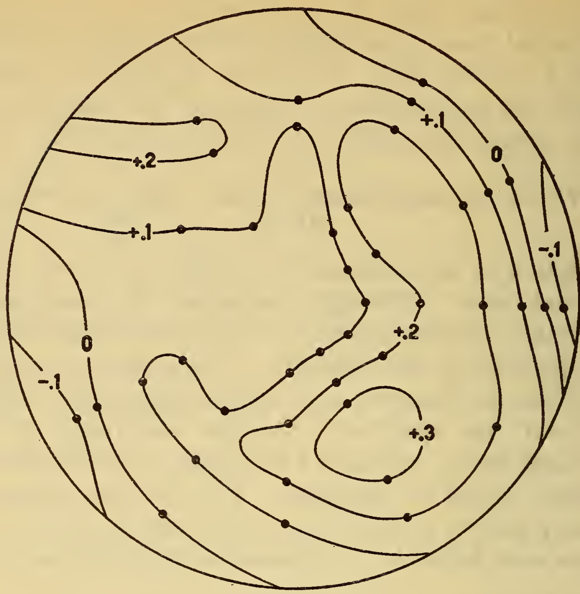
$$y = -719.7 \times 10^{-7}\ x^2 + 2.239 \times 10^{-9}\ x^4 - 2.809 \times 10^{-13}\ x^6 \qquad (14)$$

where y is the departure from flatness expressed in half-wave lengths ($\lambda = 0.5876\mu$), and x is the distance from the center of the flat in millimeters. The flat impresses twice its sagitta on the wave front after reflection; hence, the error in phase arising from the flat is

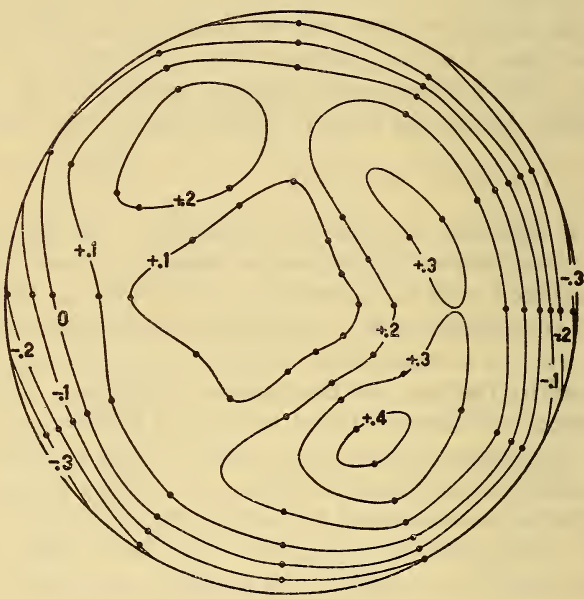
$$\eta_{\text{flat}} = -719.7 \times 10^{-7}\ x^2 + 2.239 \times 10^{-9}\ x^4 - 2.809 \times 10^{-13}\ x^6 \qquad (15)$$

where η_{flat} is expressed in whole wave lengths. For the axial measurements, η in equation (13) represents twice the phase aberration of the lens combined with the aberration of the flat, η_{flat}, and the latter is used as a correcting value. In making the oblique measurements, η, equation (13), is a combination of the axial aberration, the aberration produced by the flat, and the oblique aberration. Since the first two aberrations are known, the latter is found simply by subtraction.

A point of interest in connection with the lens is that, in testing the lens visually during its construction, the autocollimated image of an artificial star was examined by racking the eyepiece in and out of the focus. Presumably the same flat was used during these constructional tests as was used in the present investigation, and it is found that this objective contains errors which are neutralized to a certain extent by the errors of the flat. This is illustrated by Figures 10(*a*) and 10(*b*). Figure 10(*a*) shows the axial aberration at the best focus of the lens when the flat is considered perfect; while, if correction is made for the errors of the flat and the best focus again found, the contours of Figure 10(*b*) are obtained, indicating an



a



b

Fig. 10.—*Phase contours for best axial focus of objective No. 3*

a, Combined errors of lens and flat. *b*, Errors of lens only, with correction made for errors introduced by flat. This illustration indicates that the errors of lens and flat tend to neutralize each other

increase of positive phase aberration (overcorrection) near the center and at the edge an increase of the negative aberration (undercorrection). This indicates the importance of using accurate flats in testing lenses during the process of construction.

The diagram (fig. 10(*b*)), also shows the presence of a number of irregularities, from which it is to be concluded that an improvement in the performance of this objective could be effected by local and zonal retouching of the surface.

The best oblique focus, at an angular distance of 1° from the axis, is represented by Figure 11. The asymmetry from top to bottom of the figure is radial from the axis of the lens and represents a small amount of residual coma.

FIG. 11.—*Phase contours for best oblique focus of objective No. 3. (Obliquity 1°)*

IV. SUMMARY AND CONCLUSIONS

A method for measuring the axial and oblique aberrations of a lens system is described. This method, which is based on interference, determines the aberrations directly in terms of the difference of phase between the various positions of the emergent wave front in passing through any point on the axis or chief ray. By a simple computation such a point may be selected which will produce minimum phase residuals, representing the point of best imagery. The expression of the aberrations directly in terms of phase differences enables one to compare them with the Rayleigh tolerance.

The method requires only simple apparatus, but its accuracy is of the order of the geometric method of Hartmann.

Examples of the application of the method to the measurement of the monochromatic aberrations of three telescope objectives are presented. These examples show the performance of the lens as compared with the Rayleigh tolerance. The characteristic effects of the axial aberration; that is, spherical aberration; and also of the oblique aberrations, coma, and astigmatism; on the emergent wave front are illustrated.

The procedure is an accurate method, utilizing simple apparatus, for the determination of the small amounts of aberration present in astronomical objectives. It should be pointed out that even in a well-designed astronomical objective the "technical" aberrations, arising from lack of homogeneity of the glass and from errors in workmanship, are always present to some extent. The irregularities of the contours illustrated show these effects very pronouncedly. Hence, in order to correctly judge the performance of such a lens care should be taken to explore a sufficient number of points on the emergent wave surface so that reliable information can be gained concerning local irregularities, as well as the regular aberrations.

WASHINGTON, November 21, 1928.